Methods of Theoretical Physics in Cross-Disciplinary Sciences
(A survey course)

by Denis Grebenkov

14 three-hour lectures (42 hours) for master students

Short description and primary aim.

Cross-disciplinary scientific projects are getting nowadays more and more important when various problems from biology, chemistry, medicine, and economics require using methods of theoretical physics. Examples are: physics of respiration, exchange of nutrients in organs, physical models of financial markets, to name a few. The primary aim of the survey course "Methods of Theoretical Physics in Cross-Disciplinary Sciences" is to present various directions of modern science which are often beyond the scope of the classical education program. The course is started with an introductory theoretical part, in which we discuss several standard methods (e.g., scaling, eigenfunctions of elliptic operators), as well as relatively modern approaches (e.g., wavelets, fractal geometry). The main part of the course is devoted to a review of practical tasks that require using methods of theoretical physics. This survey course aims to extend scientific horizon of students, facilitate their orientation in modern cross-disciplinary fields, and find interesting topics for their further specialization during PhD.

Lecture 1. The Fractal Geometry of Nature

Physical processes at different length and time scales; various methods for their description and modeling (quantum mechanics, molecular dynamics, continuum models); notion of scaling as transition from small to large scales, difficulties; fractals as simple examples of self-similar systems; various examples of fractals: Cantor sets, Sierpinski carpet and gasket, von Koch curves and surfaces, etc., fractals from dynamical systems (Julia and Mandelbrot sets), examples of fractals in nature; special role of power laws; multifractal analysis; unsolved problems.

Bibliography:

• B. Mandelbrot, *The Fractal Geometry of Nature* (San Francisco, Freeman, 1982).

• P.-G. de Gennes, *Scaling Concepts in Polymer Physics* (Ithaca, London, Cornell University Press, 1979).

• H. E. Stanley, *Scaling, universality, and renormalization - Three pillars of modern critical phenomena*, Rev. Mod. Phys. **71**, S358 (1999).

Lecture 2. Scaling in Models of Statistical Physics

Classical models of statistical physics (various kinds of percolation, Ising model, Potts model, etc); scaling as a basic element of renormalization group theory in theory of phase transitions; universality of physical phenomena, universality classes; limiting transition from lattice models to continuous curves; Loewner stochastic evolution (SLE); unsolved problems.

Bibliography:

• R. J. Baxter, *Exactly Solved Models in Statistical Mechanics* (London, Academic Press, 1982).

• G. Parisi, *Statistical Field Theory* (Redwood City: Addison-Wesley, 1988).

• A. Erzan, L. Pietronero, A. Vespignani, *The fixed-scale transformation approach to fractal growth*, Rev. Mod. Phys. **67**, 545 (1995).

• J. Cardy, *SLE for theoretical physicists*, Annals of Physics **318**, 81 (2005).

Lecture 3. Geometry and Properties of Porous Structures

Examples of porous structures: sedimentary rocks, lungs, biological tissues (interstitium), catalysts, etc; imaging methods for observation and investigation; nuclear magnetic resonance (NMR); statistical description of porous structures; transport processes: convection and diffusion; relation between microscopic and macroscopic parameters; methods for modeling and models of porous structures; unsolved problems.

Bibliography:

• P. T. Callaghan, *Principles of Nuclear Magnetic Resonance Microscopy* (Clarendon, Oxford, 1991).

• M. Sahimi, *Flow phenomena in rocks - from continuum models to fractals, percolation, cellular automata, and simulated annealing*, Rev. Mod. Phys. **65**, 1393 (1993).

• F. A. L. Dullien, *Porous Media: Fluid Transport and Pore Structure* (Academic, New York, 1991).
• P. Z. Wong, *Methods in The Physics of Porous Media* (Academic, London, 1999).
• J. Klafter and J. M. Drake (Eds.), *Molecular Dynamics in Restricted Geometries* (John Wiley and Sons, New York, 1989).
• J. Karger and D. M. Ruthven, *Diffusion in Zeolites and Other Microporous Solids* (John Wiley & Sons, 1992).

Lecture 4. Diffusion

Diffusion as a product of random fluctuation; central limit theorem; Gaussian distribution; random walks; Brownian motion (Wiener process); mathematical basis (diffusion equation, Laplace equation, Green functions, eigenmode expansion, etc); examples in physics, chemistry, biology, physiology; unsolved problems.

Bibliography:

• P. Lévy, *Processus stochastiques et movement brownien* (Paris, Gauthier-Villard, 1965).
• W. Feller, *An Introduction to Probability Theory and Its Applications*, Volumes I and II, Second Edition (John Wiley & Sons, New York, 1971).
• R. P. Feynman, A. R. Hibbs, *Quantum Mechanics and Path Integrals* (New York, McGraw-Hill, 1965).
• F. Spitzer, *Principles of Random Walk* (New York: Springer, 1976).
• G. H. Weiss, *Aspects and Applications of the Random Walk* (North-Holland, Amsterdam, 1994).
• S. Redner, *A Guide to First-Passage Processes* (Cambridge University Press, Cambridge, England, 2001).

Lecture 5. Diffusion and Boundaries

Examples of boundaries of irregular shape; boundary value problems with Dirichlet, Neumann and Robin boundary conditions; classical methods for finding solutions; screening effect; harmonic measure; the use of multifractal analysis; Makarov theorem for two-dimensional problems and its practical applications; passivation of a surface and its consequences for heterogeneous catalysis; reflected Brownian motion, local time; spectral geometry and inverse problems; unsolved problems.

Bibliography:

• H. S. Carslaw, J. C. Jaeger, *Conduction of Heat in Solids*, 2nd Ed. (Clarendon, Oxford, 1959).
• J. Crank, *The Mathematics of Diffusion*, 2nd Ed. (Clarendon, Oxford, 1975).
• M. Freidlin, *Functional Integration and Partial Differential Equations*, Annals of Mathematics Studies (Princeton University, Princeton, New Jersey, 1985).
• B. Sapoval, *Transport Across Irregular Interfaces: Fractal Electrodes, Membranes and Catalysts*, in *Fractals and Disordered Systems*, Eds. A. Bunde, S. Havlin, pp. 233-261 (Springer-Verlag, Berlin, 1996).

Lecture 6. Eigenfunctions of the Laplace Operator and Their Applications

Definition and properties of the Laplace operator eigenfunctions; interpretation as waves of vibrating surfaces; experimental observations; examples of eigenfunctions for simple and complicated domains; statistics of eigenvalues; localization of eigenfunctions on the boundary; practical applications (noise-protective walls); using spectral methods for solving various practical problems (e.g., statistics of residence times on a subset); unsolved problems.

Bibliography:

• R. F. Bass, *Diffusions and Elliptic Operators* (Springer, 1998).
• D. S. Grebenkov, *NMR Survey of Reflected Brownian Motion*, Rev. Mod. Phys. **79**, 1077-1137 (2007).

Lecture 7. Anomalous Diffusions

Cases when the central limit theorem is not applicable, emergence of “anomalies”; Lévy process; relation to Tsallis statistics; diffusion in a medium with sinks; Knudsen diffusion (ballistic trajectories); Pearson diffusion; continuous time random walks; diffusion on fractals; fractional diffusion equation; fractional Brownian motion; unsolved problems.

Bibliography:

• J.-P. Bouchaud, A. Georges, *Anomalous diffusion in disordered media: Statistical mechanisms, models and physical applications*, Phys. Rep. **195**, 127-293 (1990).
• J. W. Haus, K. W. Kehr, *Diffusion in regular and disordered lattices*, Phys. Rep. **150**, 263-406 (1987).
• S. Havlin, D. ben Avraham, *Diffusion in disordered media*, Adv. Phys. **51**, 187-292 (2002).
• G. M. Zaslavsky, *Chaos, fractional kinetics, and anomalous transport*, Phys. Rep. **371**, 461-580 (2002).

• M.F. Shlesinger, J. Klafter, G. Zumofen, *Above, below and beyond Brownian motion*, Am. J. Phys. **67**, 1253 (1999).
• J. Klafter, M. F. Shlesinger, G. Zumofen, *Beyond Brownian Motion*, Physics Today, February 33 (1996).
• M. F. Shlesinger, G. M. Zaslavsky, J. Klafter, *Strange kinetics*, Nature **363**, 31 (1993).

Lecture 8. Processes of Diffusive Growth

Examples of diffusive growth in nature: Hele-Show growth between two liquids of different viscosity, irreversible colloidal aggregation (Smoluchowski coagulation), dendric growthin electrochemistry, etc.; simple model of diffusion-limited aggregation (DLA), discovered properties and open questions; formation of instabilities on the boundary; unsolved problems.

Bibliography:

• T. C. Halsey, *Diffusion-limited aggregation: A model for pattern formation*, Physics Today **53**, 36-41 (2000)
• T. A. Witten, L. M. Sander, *Diffusion-Limited Aggregation, a Kinetic Critical Phenomenon*, Phys. Rev. Lett. **47**, 1400 (1981).
• P. Bak, C. Tang, K. Wiesenfeld, *Self-Organized Criticality - An Explanation of 1-f Noise*, Phys. Rev. Lett. **59**, 381 (1987).
• J. S. Langer, *Instabilities and pattern formation in crystal growth*, Rev. Mod. Phys. **52**, 1 (1980)
• M. C. Cross, P. C. Hohenberg, *Pattern formation outside of equilibrium*, Rev. Mod. Phys. **65**, 851 (1993).
• A. J. Koch, H. Meinhardt, *Biological pattern formation - from basic mechanisms to complex structures*, Rev. Mod. Phys. **66**, 1481 (1994).
• C. Bowman, A. C. Newell, *Natural patterns and wavelets*, Rev. Mod. Phys. **70**, 289 (1998).
• J. P. Gollub, J. S. Langer, *Pattern formation in nonequilibrium physics*, Rev. Mod. Phys. **71**, S396 (1999).

Lecture 9. Physics of Respiration

Geometric structure of the lungs; basic facts about respiration physiology; elements of hydrodynamics in branching structures (bronchial tree); diffusive transport of oxygen and carbon dioxide across alveolar membranes; screening effect; various functioning modes of the lungs; optimal functioning; unsolved problems.

Bibliography:

• E. R. Weibel, *The Pathway for oxygen. Structure and function in the mammalian respiratory system* (Harvard University, Cambridge, Massachusetts and London, England, 1984).
• B. Mauroy, M. Filoche, E. Weibel, B. Sapoval, *An Optimal Bronchial Tree May Be Dangerous*, Nature **427**, 633 (2004).
• M. Felici, M. Filoche, B. Sapoval, *Renormalized Random Walk Study of Oxygen Absorption in the Human Lung*, Phys. Rev. Lett. **92**, 068101 (2004).
• W. R. Hendee, *Physics and applications of medical imaging*, Rev. Mod. Phys. **71**, S444 (1999).

Lecture 10. Statistical Physics of Networks

Examples of various networks: internet, social networks, citation networks, transportation networks (airports), genome networks, genealogical trees; Statistical physics of networks, power law distributions, free-scale networks. Models of growing networks: random networks (Erdos-Renyi); model of preferential attachment (Barabasi, Albert), model of local attachment (Sherrington); unsolved problems.

Bibliography:

• R. Albert, A.-L. Barabasi, *Statistical mechanics of complex networks*, Rev. Mod. Phys. **74**, 47 (2002).
• R. Albert, H. Jeong, A.-L. Barabasi, *Diameter of the World-Wide Web*, Nature **401**, 130 (1999).
• T. Nakayama, K. Yakubo, R. L. Orbach, *Dynamical properties of fractal networks - Scaling, numerical simulations, and physical realizations*, Rev. Mod. Phys. **66**, 381 (1994).

Lecture 11. The Granular World

Granular media as examples of small systems, in which classical mechanics is already unefficient while statistical physics is not yet applicable. Examples of various phenomena: segregation (Brazil nut effects), crystallization, jamming, intermittent motion, formation of instabilities. Models of hard and soft spheres, phase transitions. Elements of theoretical description (Edwards’ approach). Modeling by methods of molecular dynamics; unsolved problems.

Bibliography:

• H. M. Jaeger, S. R. Nagel, R. P. Behringer, *Granular solids, liquids, and gases*, Rev. Mod. Phys. **68**, 1259 (1996).
• I. S. Aranson, L. S. Tsimring, *Patterns and collective behavior in granular media - Theoretical concepts*, Rev. Mod. Phys. **78**, 641 (2006).
• P.-G. de Gennes, *Granular matter - a tentative view*, Rev. Mod. Phys. **71**, S374 (1999).

Lecture 12. Applications in Computer Sciences

Discrete form of the Laplace operator for images; diffusion equation and variational principle for image processing; automated edge detection on the image; diffusion on the graphs (e.g., internet) and discrete Laplace operator for fast searching (principle of Google); methods for sorting; unsolved problems.

Bibliography:

• R. R. Coifman, S. Lafon, A. B. Lee, M. Maggioni, B. Nadler, F. Warner, S. W. Zucker, *Geometric diffusions as a tool for harmonic analysis and structure definition of data - Diffusion maps*, PNAS **102**, 7426 (2005).
• R. R. Coifman, S. Lafon, A. B. Lee, M. Maggioni, B. Nadler, F. Warner, S. W. Zucker, *Geometric diffusions as a tool for harmonic analysis and structure definition of data - Multiscale methods*, PNAS **102**, 7432 (2005).

Lecture 13. Wavelets and Their Applications

Difficulty of modeling mineral and biological structures (multiscale); problem of upscaling – transition from small scales (microscopic description) to large scale (macroscopic description); notion of wavelets; using wavelets for describing and modeling porous structures; image compressing; explicit construction of Brownian motion; unsolved problems.

Bibliography:

• S. Mallat, *A Wavelet Tour of Signal Processing* (Academic Press, 2nd edition, 1999).

Lecture 14. Elements of Numerical Modeling

Survey of methods for numerical modeling at different time and length scales; calculation *ab initio* (quantum description); molecular dynamics; Monte Carlo; continuum models (diffusion, hydrodynamics); methods of finite elements and finite differences (engineering approaches); method of phase field for description of moving boundaries; unsolved problems.

Bibliography:

• K. K. Sabelfeld, *Monte Carlo Methods in Boundary Value Problems* (Springer-Verlag: New York - Heidelberg, Berlin, 1991).
• K. K. Sabelfeld and N. A. Simonov, *Random Walks on Boundary for Solving PDEs* (Utrecht, The Netherlands, 1994).
• G. N. Milshtein, *Numerical Integration of Stochastic Differential Equations* (Kluwer, Dordrecht, the Netherlands, 1995).
• D. M. Ceperley, *Microscopic simulations in physics*, Rev. Mod. Phys. **71**, S438 (1999).