Modelling of the Respiratory System and Diffusion-Weighted Imaging

Denis S. Grebenkov

Laboratoire de Physique de la Matière Condensée,
CNRS -- Ecole Polytechnique, F-91128 Palaiseau, France
denis.grebenkov@polytechnique.edu

In the second part of the lecture "Modelling of the Respiratory System", we focus on several geometrical and physiological models which are relevant for diffusion-weighted imaging of the lungs [1-6]. We start by recalling how inhomogeneous time-dependent magnetic fields encode random trajectories of the nuclei (net dephasing in a linear gradient, Bloch-Torrey equation, Gaussian form of the spin-echo signal for free diffusion, etc.) [7]. The effects of geometrical confinement on diffusive motion and the consequent signal attenuation are then considered [8]. As the simplest and broadly used frame, the notion of apparent diffusion coefficient (ADC) is introduced. The limitations of ADC and of the related b-value are explained. Further extension towards diffusional kurtosis imaging [9] is briefly discussed. Biexponential and stretch-exponential fitting formulas are also mentioned [10]. A special attention is paid to two particular models. In the first model, the alveolar ducts (or airways) are approximated by long cylinders, either smooth or covered with alveolar sleeves [11]. For relatively short diffusion times, the branching structure of the acinus is neglected, allowing one to deduce a simple analytical formula for the signal attenuation. The underlying longitudinal and transverse diffusion coefficients are related to geometrical parameters of this cylinder model. The advantages and inconveniences of the model are interrogated [12,13]. At longer diffusion times, which are currently accessible in experiments by using stimulated spin-echo techniques [14], the acinar branching structure is expected to play an important role. These and many other issues require a more realistic geometrical model of the acinus. The Kitaoka algorithm allows one to generate three-dimensional branching labyrinths densely filling a given volume [15]. On one hand, these structures present geometrical properties that are very similar to real acini (total surface area, surface-to-volume ratio, average length of branches, etc.). On the other hand, the fully controlled generating procedure makes these structures particularly suitable for numerical modelling [16-17]. The use of the Kitaoka model is illustrated by Monte Carlo simulations of restricted diffusion of the hyperpolarized helium-3 in the model healthy and emphysematous acini [18].

References

[1] B. T. Saam, D. A. Yablonskiy, V. D. Kodibagkar, J. C. Leawoods, D. S. Gierada, J. D. Cooper, S. S. Lefrak, and M. S. Conradi, *MR imaging of diffusion of 3He gas in healthy and diseased lungs*, Magn. Reson. Med. **44**, 174 (2000).

[2] H. E. Moller, X. J. Chen, B. Saam, K. D. Hagspiel, G. A. Johnson, T. A. Altes, E. E. de Lange, and H.-U. Kauczor, *MRI of the lungs using hyperpolarized noble gases*, Magn. Reson. Med. **47**, 1029 (2002).

[3] E. J. R. van Beek, J. M. Wild, H.-U. Kauczor, W. Schreiber, J. P. Mugler, and E. E. de Lange, *Functional MRI of the Lung Using Hyperpolarized 3-Helium Gas*, J. Magn. Reson. Imag. **20**, 540 (2004).

[4] A. J. Swift, J. M. Wild, S. Fichele, N. Woodhouse, S. Fleming, J. Waterhouse, R. A. Lawson, M. N. J. Paley, E. J. R. van Beek, *Emphysematous changes and normal variation in smokers and COPD patients using diffusion 3He MRI*, Eur. J. Radiol. **54**, 352 (2005).

[5] S. J. Kadlecek, K. Emami, M. C. Fischer, M. Ishii, J. Yu, J. M. Woodburn, M. Nikkhah, V. Vahdat, D. A. Lipson, J. E. Baumgardner, and R. R. Rizi, *Imaging physiological parameters with hyperpolarized gas MRI*, Progr. Nucl. Magn. Reson. Spectr. **47**, 187 (2005).

[6] M. S. Conradi, B. Saam, D. A. Yablonskiy, and J. C. Woods, *Hyperpolarized 3He and perfluorocarbon gas diffusion MRI of lungs*, Progr. Nucl. Magn. Reson. Spectr. **48**, 63 (2006).

[7] P. T. Callaghan, *Principles of Nuclear Magnetic Resonance Microscopy* (Clarendon Press, Oxford, 1991).

[8] D. S. Grebenkov, *NMR Survey of Reflected Brownian Motion*, Rev. Mod. Phys. **79**, 1077 (2007).

[9] R. Trampel, J. H. Jensen, R. F. Lee, I. Kamenetskiy, G. McGuinness, and G. Johnson, *Diffusional Kurtosis Imaging in the Lung Using Hyperpolarized 3He*, Magn. Reson. Med. **56**, 733 (2006).

[10] V. G. Kiselev and K. A. Il'yasov, *Is the 'Biexponential Diffusion' Biexponential?*, Magn. Reson. Med. **57**, 464 (2007).

[11] D. A. Yablonskiy, A. L. Sukstanskii, J. C. Leawoods, D. S. Gierada, G. L. Bretthorst, S. S. Lefrak, J. D. Cooper, and M. S. Conradi, *Quantitative in vivo Assessment of Lung Microstructure at the Alveolar Level with Hyperpolarized 3He Diffusion MRI*, Proc. Nat. Ac. Sc. **99,** 3111 (2002).

[12] S. Fichele, M. N. J. Paley, N. Woodhouse, P. D. Griffiths, E. J. R. van Beek, and J. M. Wild, *Finite-Difference Simulations of 3He Diffusion in 3D Alveolar Ducts: Comparison with the `Cylinder Model'*, Magn. Res. Med. **52**, 917 (2004).

[13] R. E. Jacob, G. Laicher, and K. R. Minard, *3D MRI of non-Gaussian 3He gas diffusion in the rat lung*, J. Magn. Reson. **188**, 357 (2007).

[14] C. Wang, G. W. Miller, T. A. Altes, E. E de Lange, G. D. Cates, J. P. Mugler III, *Time Dependence of 3He Diffusion in the Human Lung - Measurement in the Long-Time Regime Using Stimulated Echoes*, Magn. Reson. Med. **56**, 296 (2006).

[15] H. Kitaoka, S. Tamura, R. Takaki, *A Three-Dimensional Model of the Human Pulmonary Acinus*, J. Appl. Physiol. **88**, 2260 (2000).

[16] M. Felici, M. Filoche, C. Straus, T. Similowski, and B. Sapoval, *Diffusional Screening in Real 3D Human Acini -- a Theoretical Study*, Resp. Physiol. Neurobiol. **145**, 279 (2005).

[17] D. S. Grebenkov, M. Filoche, B. Sapoval, and M. Felici, *Diffusion-Reaction in Branched Structures: Theory and Application to the Lung Acinus*, Phys. Rev. Lett. **94**, 050602 (2005).

[18] D. S. Grebenkov, G. Guillot, and B. Sapoval, *Restricted Diffusion in a Model Acinar Labyrinth by NMR. Theoretical and Numerical Results*, J. Magn. Reson. **184**, 143 (2007).