Scaling Properties of the Spread Harmonic Measures

by Denis Grebenkov

Diffusive transport across a semi-permeable interface is ubiquitous in physics, biology, chemistry and industry [1,2]. Diffusing particles first arrive onto the boundary and then explore some neighboring area before reacting or being transferred across this interface. The overall functioning of the interface is then governed by a "competition" between its accessibility (how easy to reach the boundary) and permeability (how easy to cross the boundary). This competition is controlled by two transport parameters: diffusion coefficient D and surface permeability or reactivity W. Their ratio D/W, which is homogeneous to a length, parameterizes a transition from a purely reactive boundary ($W = \infty$) and a purely reflecting boundary ($W = 0$).

In mathematical terms, the arrival points for diffusion are known to be characterized by harmonic measure. By analogy, one can introduce a family of the spread harmonic measures (parameterized by the ratio D/W) in order to characterize the transfer or reaction points [3-5]. These measures can either be generated by partially reflected Brownian motion, or obtained as solutions of the related boundary value problems for Laplace operator [4-5].

A numerical analysis of the spread harmonic measures on prefractal quadratic von Koch curves has revealed many interesting properties, some of them having remained poorly understood in a rigorous mathematical sense. For instance, the family of the spread harmonic measures exhibits a transition between the harmonic measure ($D/W = 0$) and the Hausdorff measure ($D/W \to \infty$). To our knowledge, this is the first numerical observation of such a transition between two measures having so different multifractal behaviors on fractal sets. Scaling properties of the spread harmonic measures on prefractal boundaries are found to be characterized by a set of multifractal exponent functions depending on the only scaling parameter. A conjectural extension of the spread harmonic measures to fractal boundaries is proposed.

References

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